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IMPACT OF STRONG MOLECULE THICKNESS ON SHOCK STRENGTH, CYLINDRICAL SEPARATION, AND FLOW CHARACTERISTICS IN THE PRESENCE OF AN AZIMUTHAL MAGNETIC FIELD.

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Abstract
A model that is self-similar for one-dimensional unsteady isothermal flows in a rotating atmosphere is presented, where the shock wave (SW) is generated by a cylindrical piston that moves with time according to an exponential law. The ideal gas is subject to an "azimuthal magnetic field (AMF) and has variable density". Theoretically, a SW that is diverging should be moving away from the axis of symmetry. The fluid's axial and azimuthal components are presumed to be changeable in the surrounding medium. It is assumed that the surrounding medium's fluid velocities, starting density, and initial magnetic field are dynamic and subject to power laws. Assuming a perfectly symmetrical gas that rotates with a constant intensity, they model the radiation flux passing through it and the energy absorption occurring solely behind the SW, which travels in the reverse direction as the radiation flow. Changes to the Alfven-Mach number and diabatic exponent are studied for their consequences. SWs exhibit diminishing effects when a magnetic field is present, as has been discovered. Additionally, it is noted that in the case of adiabatic flow, the impact of a stronger magnetic field is more pronounced compared to isothermal flow.

Keywords: Azimuthal magnetic field, Shock wave, Self-similar model, cylindrical shock wave (CSW)

1. Introduction

The ionization of gases across the shock's target area and the medium's subsequent behavior consistent with a very electrically conductive medium are both consequences of the high temperatures typically experienced during SW propagation. Therefore, gas-dynamic flow and electromagnetic theory should be studied concurrently for a comprehensive examination of this type of problem. It is critical to investigate the transmission of CSWs in "conducting gas in the circumstance of an axial or AMF while performing tests on the pinch effect, bursting wires, and so on. Analyzing the connection of the magnetic field with the other flow variables is critical because the material inside a star is a plasma with infinite electrical conductivity that exists within a strong magnetic field" [1].

The interplay of various physical parameters in complex systems often leads to intricate and fascinating phenomena. In this context, the research delves into the intriguing dynamics of strong molecule thickness and the presence of an AMF on shock strength, cylindrical separation, and flow characteristics within a medium. Understanding the ramifications of these interactions is vital for a multitude of applications in fields ranging from astrophysics and aerospace engineering to plasma physics. The strength of SWs is a critical parameter in many natural and engineered systems. It directly impacts the energy transfer, turbulence, and compression within the medium. In this study, they aim to explore how variations in strong molecule thickness affect the shock strength, shedding light on the underlying mechanisms governing shock dynamics.

The goal of this study "is to find the solutions that are self-similar to the flow behind the CSW that a moving piston creates in a non-ideal gas flowing in a rotational axisymmetric flow with a variable

AMF and a variable axial component of the fluid velocity, all while maintaining isothermal flow conditions given by Nath" [2,3].
As a result, "the one-dimensional unsteady self-similar rotational axisymmetric flow of a non-ideal gas behind a shock that is emitted by a cylinder piston that moves exponentially in time while being subjected to a magnetic field is the subject of this investigation. It is assumed that the exponential law proposed by Ranga Rao and Ramanna governs the piston's movement"[4].

$$r_p = C\exp(\sigma t),\ \sigma > 0 \tag{1}$$

Within this particular framework, "σ represents a constant with dimensions, r_p denotes the radius of the piston, and 'C' signifies the initial radius of the piston". Moreover, the variable t symbolizes the concept of time.
Additionally, the exponential law should be followed by the shock propagation.

$$r_s = D\exp(\sigma t) \tag{2}$$

The shock radius, denoted by r_s, and the dimensional constant by D. The value of D is dependent on the 'C' piston's non-dimensional location.
In this context, the isothermal flow that is being considered is physically realistic. The solutions are shown in Sections 2 and 3, with the assumption that the flow is isothermal. The effects of various parameters like "the ratio of the gas's specific heats, the gas's non-idealness, and the Alfven-Mach number are examined in relation to the flow variables and the shock strength. It is in section 4 that they discover the solutions to the problem based on isothermal flows". Additionally, the conclusion of the research is presented in the fifth portion of the paper.

1.1 SWs: A Fundamental Parameter

SWs are a crucial concept in fluid dynamics, playing a fundamental role in a wide range of scientific and engineering applications. Understanding the significance of SWs in fluid dynamics and their practical implications is essential for many fields. Here's an explanation of their importance:

1) **Energy Transfer and Compression:** SWs are abrupt, high-energy disturbances that travel through a fluid medium, compressing and transferring energy. This compression and energy transfer are pivotal in a variety of situations, such as supersonic and hypersonic flight, explosions, and industrial processes. In aerodynamics, for example, SWs are responsible for the compression of air and the resulting increase in temperature and pressure, which has significant implications for aircraft design and performance
2) **Turbulence and Mixing:** SWs can induce turbulence and mixing in fluids. When a SW interacts with a boundary or another SW, it can create turbulent flow patterns. These turbulent flows can influence the dispersion of particles and the mixing of materials in a fluid, impacting processes like combustion in engines, pollutant dispersion in the atmosphere, and even the behavior of supernovae in astrophysics.
3) **Astronomical Phenomena:** In astrophysics, SWs are commonly observed in various celestial events, including supernovae, pulsar wind nebulae, and the formation of galaxies and stars. They play a vital role in redistributing energy and matter in space, influencing the dynamics of the universe.
4) **Biomedical and Medical Applications:** SWs are used in medical procedures, such as lithotripsy, for breaking down kidney stones without invasive surgery. Understanding how SWs propagate through tissues is crucial in improving the efficiency and safety of these medical treatments.
5) **Explosive Detonation and Blast Waves:** In military and defense applications, understanding the behavior of SWs is essential for designing protective structures, assessing the effects of explosions, and optimizing the performance of explosive devices.
6) **Material Science and High-Pressure Studies:** SWs can be used to study material properties at extreme pressures. This is valuable for research in materials science and geophysics to understand the behavior of materials under extreme conditions.
7) **Aerospace Engineering:** The study of SWs is indispensable for the design of supersonic and hypersonic aircraft and spacecraft. Efficiently controlling and mitigating SWs can lead to

breakthroughs in aerospace engineering, making high-speed travel more practical and cost-effective.

2. Equations fundamentals and boundary conditions

The gas "dynamics equations in Eulerian coordinates that describe the motion of rotational axisymmetric non-ideal gas in a one-dimensional isothermal flow as a function of a gravitational field", an AMF or axial magnetic field, and other external fields can be stated as [5-8]

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\frac{\partial u}{\partial r}+\frac{u\rho}{r}=0 \tag{1}$$

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial r}+\frac{\partial p}{\partial r}+\frac{\partial h}{\partial r}+\frac{2ih}{r}-\frac{\rho v^2}{r}+\frac{m\rho G}{r}=0 \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{uv}{r}=0 \tag{3}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}=0 \tag{4}$$

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial r}+2h\frac{\partial u}{\partial r}+\frac{2hu(1-i)}{r}=0 \tag{5}$$

$$\frac{\partial m}{\partial r}-2\pi\rho r=0 \tag{6}$$

$$\frac{\partial T}{\partial r}=0 \tag{7}$$

In the given equation, "$t\ and\ r$ are time and space coordinates that are not related to each other. The variables $p, \rho, u, and\ w$ represent the pressure, density, radial, azimuthal, and axial components of the fluid velocity $\overrightarrow{V}$ in the cylindrical coordinates (r, θ, z). The variables $h, \mu, and\ T$ are for the AMF, magnetic permeability, and temperature, respectively. In this case, they believe the gas's electrical conductivity to be limitless" [9].

Also,

$$v = Ar,$$

In above equation, "A stands for the medium's angular velocity at a radial distance r from the symmetry axis". Here, the vector of vorticity

$$\vec{\zeta}=\frac{1}{2}Curl\vec{q} \tag{8}$$

has the components

$$\zeta_r=0, \zeta_\theta=-\frac{1}{2}\frac{\partial w}{\partial r}, \zeta_z=\frac{1}{2}\frac{\partial}{\partial r}(rv) \tag{9}$$

If "the thermal energy, E_t, i.e.$E_i \ll E_t$, is significantly greater than the average particle interaction energy, E_i, then After that, they assert that the gas is flawless. When the particle-particle contact is tiny or when the gas is sufficiently rarefied, this condition is satisfied. The ideal gas hypothesis is generally an acceptable choice in many astrophysical contexts[10]. Figure 1 shows the directions in which the components of the velocity vector".

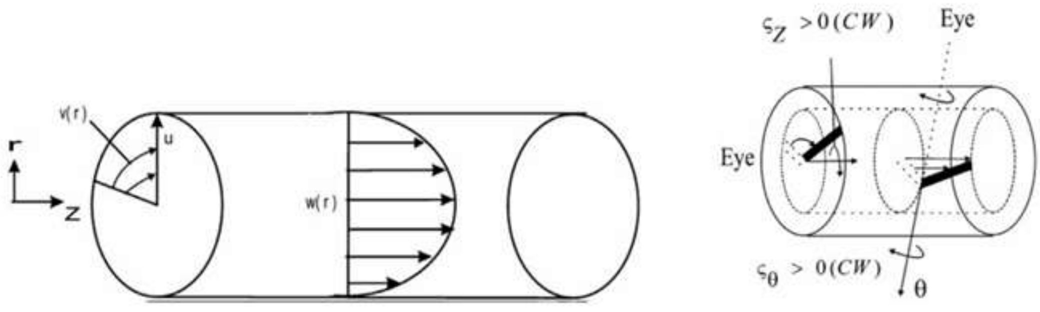


Figure 1: The velocity vector's components' directions

It is recommended that "an equation of state (1-7) be added to the system". An assumption is made about the medium's behavior as an ideal gas, such that [11,12]

$$p=\Gamma\rho T; e=C_vT=\frac{p}{(\gamma-1)\rho} \tag{10}$$

According to the equation, "C represents the gas constant, T represents the temperature, $C_v=\frac{\Gamma}{\gamma-1}$ represents the specific heat at constant volume, and γ represents the ratio of specific heats".

“A powerful CSW is meant to be moving through an ideal gas that has not been disturbed and has a variable density. When there is an AMF with a zero-radial-velocity and variable-axial-velocity configuration, this takes place. Just in front of the shock front are these flow variables”:

$$u = u_a = 0 \tag{11}$$
$$\rho = \rho_a = \rho_0 \exp(-\sigma t),\ \sigma > 0 \tag{12}$$
$$v = v_a = C \exp(\delta t) \tag{13}$$
$$w = w_a = E \exp(\alpha t) \tag{14}$$
$$h = h_a = h_0 \exp(-\lambda t) \tag{15}$$

The dimensional constants are denoted by the letters $\rho_0, C, E, h_0, \sigma, \delta, \alpha$, whereas the subscript 'a' specifically denotes to the circumstances that are immediately before the shock front.

3. Self-similarity transformations

Similar solutions could be obtained by expressing the flow pattern's field variables as dimensionless functions of ξ [13-15]

$u = WU(\xi),\ v - W\phi(\xi),\ w = WV(\xi),\ \rho =$ρ0$u = WU(\xi),\ v = W\phi(\xi),\ w = WV(\xi),\ \rho = \rho_0 G(\xi)$

$$p = \rho_0 W^2 P(\xi),\ \sqrt{\mu}h = \sqrt{\rho_0} WH(\xi),\ e = W^2 E(\xi),\ j = j_0 J(n) \tag{16}$$

U,$\phi, V, G, P, H, and\ E$ are all functions of ξ only, the dimensionless quantity is $\xi = \frac{r}{R}$ [16,17].

It is necessary for M and M_A to be constants in order for similarity solutions to exist; therefore,

$$-\lambda = 2\delta + 2, and\ \lambda = 2\sigma \tag{17}$$

Thus,

$$M^2 = \gamma^{-1}\left[-\frac{1}{2} + \frac{\rho_0 C_a^2}{(2\delta+2)\mu h_a^2}\right]^{-1} M_A^{-2} \tag{18}$$

It is possible to convert and simplify equation 18 by making use of similarity transformations from equation 16.

$$(U-\xi)\frac{dG}{d\xi} + G\frac{dU}{d\xi} + \frac{GU}{\xi} = 0 \tag{19}$$
$$(U-\xi)\frac{dU}{d\xi} - \frac{\lambda U}{2} + \frac{1}{G}\left[\frac{dP}{d\xi} + H\frac{dH}{d\xi}\right] - \frac{\phi^2}{\xi} = 0 \tag{20}$$
$$(U-\xi)\frac{d\phi}{d\xi} - \frac{\lambda}{2}V = 0 \tag{21}$$
$$\frac{dU}{d\xi}H + (U-\xi)\frac{dH}{d\xi} + \frac{UH}{\xi} - \frac{\lambda H}{2} = 0 \tag{22}$$

By using the similarity transformations (19-22), the shock conditions (1-7), which were turned into [18]

$$\frac{dU}{d\xi} = -\frac{(U-\xi)}{G}L\theta - \frac{U}{\xi} + \frac{\sigma}{i} \tag{23}$$
$$\frac{dG}{d\xi} = L\theta \tag{24}$$
$$\frac{dH}{d\xi} = \frac{H}{G}L\theta + \frac{H}{\xi} - \frac{\sigma H}{2i(U-\xi)} \tag{25}$$
$$\frac{d\phi}{d\xi} = -\frac{\phi(U+\xi)}{\xi(U-\xi)} \tag{26}$$
$$\frac{dW}{d\xi} = -\frac{W}{(U-\xi)} \tag{27}$$

Where

$$L = L(\xi) = \left[\frac{U^2}{\xi} - (U-\xi)\frac{\sigma}{i} - \frac{2H^2}{G\xi} + \frac{\sigma H^2}{2iG(U-\xi)} - 2U + \frac{\phi^2}{\xi}\right] \tag{28}$$

and

$$\theta = \theta(\xi) = \frac{2G^2\beta}{2G\beta^2(1-\beta) + GM_A^{-2}(\beta^2-1) + 2\beta H^2 - 2G\beta(U-\xi)^2}$$

They derived “the non-dimensional components of the velocity vector in the flow-filed after the shock by applying similarity transformations”. These components are $l_r = \frac{\zeta_r}{V/r_s}, l_\theta = \frac{\zeta_\theta}{V/r_s}, l_z = \frac{\zeta_z}{V/r_s}$.

$l_r = 0$

$l_\theta = \frac{W}{2(U-\xi)}$

$l_z = -\frac{\phi}{(U-\xi)}$

The shock condition can be used to determine "the boundary condition at the strong shock front" (1-7), which indicates that

$G(1) = \frac{1}{\beta}$

$U(1) = (1-\beta)$

$P(1) = \left[(1-\beta) + \frac{M_A^{-2}}{2}\left(\frac{1}{\beta^2}\right)\right]$

$\phi(1) = \frac{C}{i\eta}$

$W(1) = \frac{E}{i\eta}$

$H(1) = \frac{1}{\beta M_A}$ (29)

Thus, "it is essential to use the equation $\delta = i = \alpha$ to get the answer for similarity. The condition that must be met at the surface of the piston, in addition to the shock requirements, is that the fluid's velocity must be the same as the piston's velocity. The condition that must be met is this. This kinematic condition at the piston face could be expressed in a non-dimensional form as"

$$U(\xi_p) = \xi_p \quad (30)$$

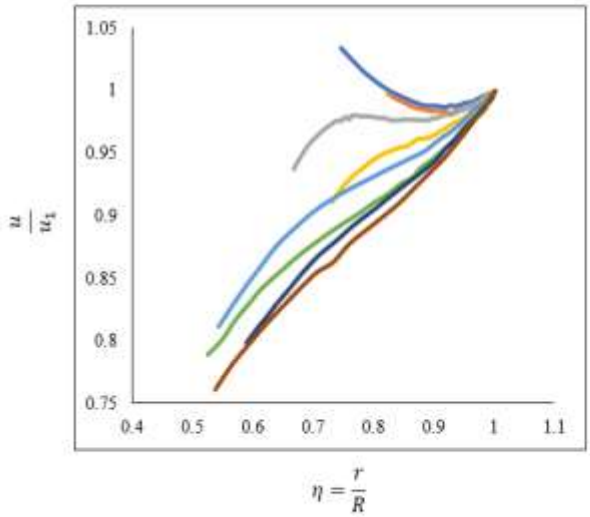
4. Result and Discussion

Using the fourth-order Runge-Kutta method, numerically integrating the set of differential equations (23)-(27) with the boundary conditions (29) and (30) yields the distributions of the flow variables in the flow field behind the shock front. 'Mathematica' software is employed for numerical integration, with a default setting of one thousand steps. "This ensures that the step size (h) is equal to the distance between a neighboring point and the inner expanding surface or the shock front, divided by one thousand. As an example, consider curve 1 in Figure 2, where $h = 1.7945 \times 10^{-4}$. When using the Runge–Kutta method of order four, the value of the interpolating function is accurate up to the first four powers of h". However, the approach contains errors in the order of h^5. When it comes to the fourth-order Runge–Kutta formula, they highly recommend that readers consult Vishwakarma and Nath [19]. For "mathematical integration, the values of the constant parameters are taken to be $\gamma = \frac{4}{3}, \frac{5}{3}$; $M_A^{-2} = 0, 0.01, 0.02, 0.05, 0.1; and\ t/t_0 = 1,2$ [20-25]. When considering completely ionized gas with $\gamma = 5/3$ and relativistic gas with $\gamma = 4/3$, it is important to note that these two values of c represent the broadest range of values seen in actual stars. Therefore, it applies to stellar medium. As shown by Rosenau and Frankenthal [26], the effects of the magnetic field on the flow field behind the SW are deemed to be substantial when the value of M_A^{-2} is more than or equal to 0.01. Consequently, the values of M_A^{-2} provided above are used for computation in the current problem".

In the non-magnetic scenario, the value $M_A^{-2} = 0$ stands out as the appropriate value. The current study is an "allowance of the work that Nath and Nath have done by consideration the rotation of the medium as well as the component of the vorticity vector (see Figure 2(C), (F), (H), and (I))".

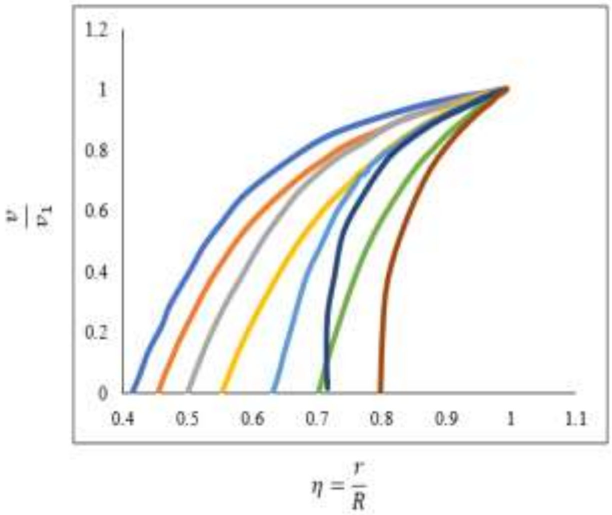
There are two tables that display the variation in the density ratio $\beta(=\rho_1/\rho_2)$ throughout the shock front and the location of the inner expanding surface. Table I displays the variation for various values of M_A^{-2} with $\gamma = 5/3$, while Table II displays the variation for dissimilar values of M_A^{-2} and γ with ${}^{t}/{}_{t_0} = 1,1.5,2$.

The flow variables $\frac{u}{u_1}, \frac{v}{v_1}, \frac{w}{w_1}, \frac{p}{p_1}, \frac{\rho}{\rho_1}, \frac{h}{h_1}, \frac{j}{j_1}$ are depicted in Figure 2(A)–(I). Additionally, "the non-dimensional azimuthal component of the vorticity vector l_θ and the non-dimensional axial component of the vorticity vector l_z are shown in relation to the similarity variable η at different values of the parameters M_A^{-2} and γ".



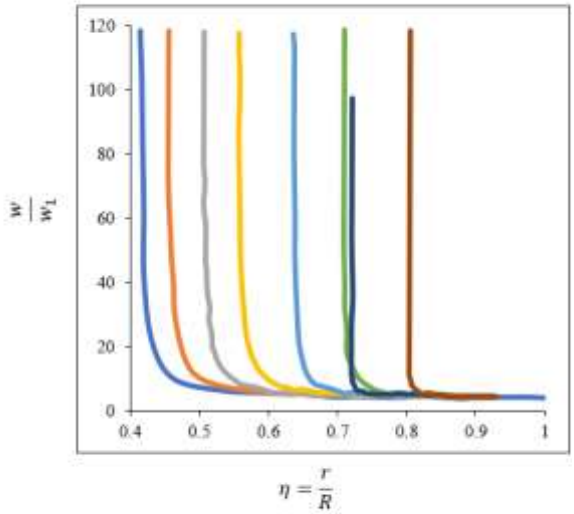
1.05
1
0.95
0.9
0.85
0.8
0.75
0.4 0.5 0.6 0.7 0.8 0.9 1 1.1
u/u_1
η = r/R

(A)



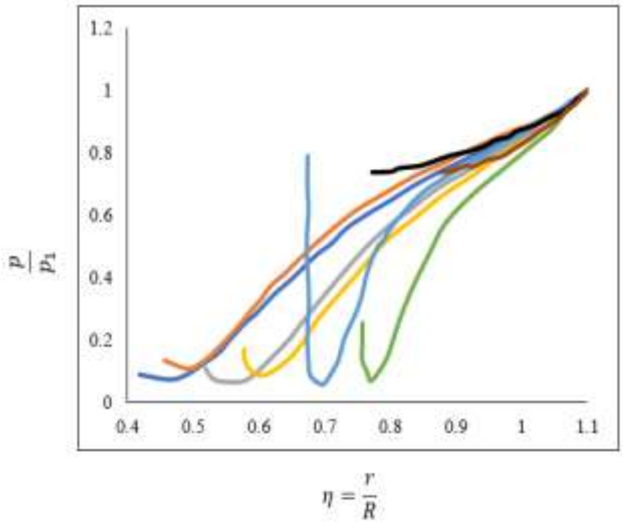
1.2
1
0.8
0.6
0.4
0.2
0
0.4 0.5 0.6 0.7 0.8 0.9 1 1.1
v/v_1
η = r/R

(B)



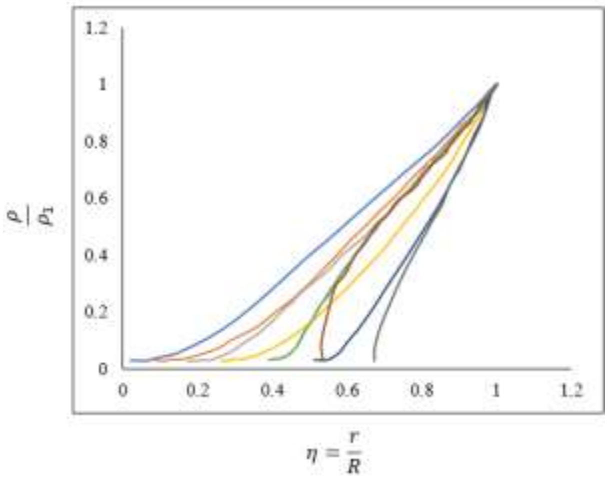
120
100
80
60
40
20
0
0.4 0.5 0.6 0.7 0.8 0.9 1
w/w_1
η = r/R

(C)



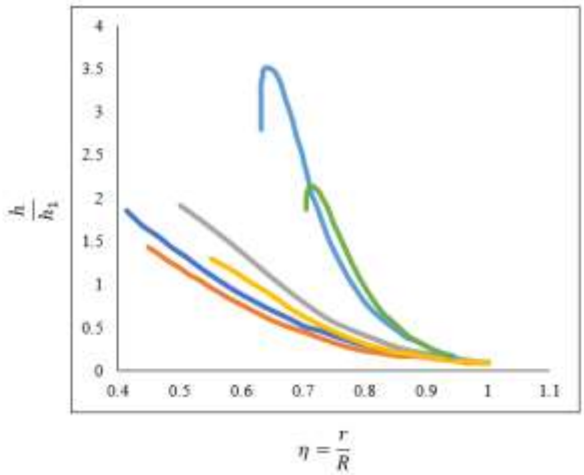
1.2
1
0.8
0.6
0.4
0.2
0
0.4 0.5 0.6 0.7 0.8 0.9 1 1.1
p/p_1
η = r/R

(D)



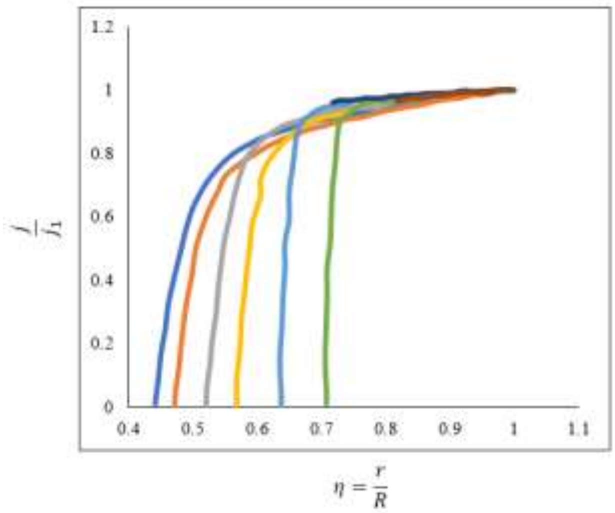
1.2
1
0.8
0.6
0.4
0.2
0
0 0.2 0.4 0.6 0.8 1 1.2
ρ/ρ_1
η = r/R

(E)

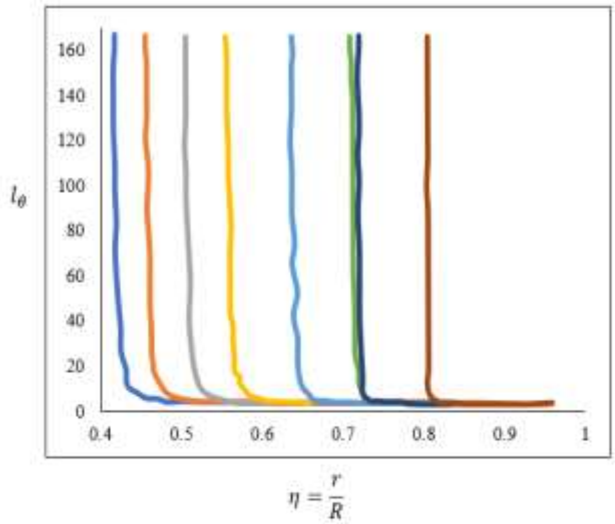


4
3.5
3
2.5
2
1.5
1
0.5
0
0.4 0.5 0.6 0.7 0.8 0.9 1 1.1
h/h_1
η = r/R

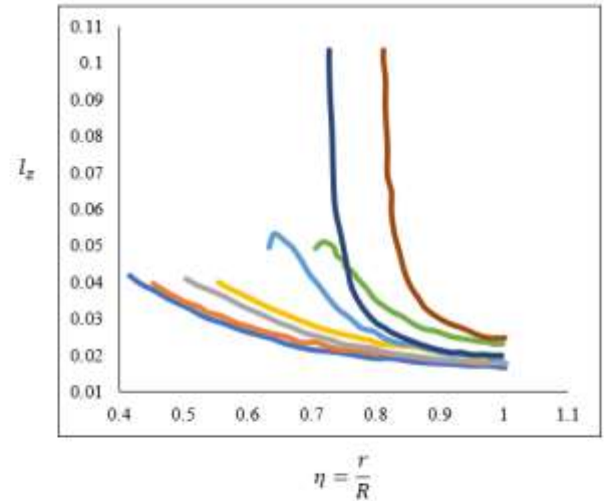
(F)



(G)



(H)



(I)

Figure 2: "Reduced flow changeable variation behind the shock front (A) Radial velocity, (B) azimuthal velocity, (C) axial velocity, (D) pressure, (E) density, (F) AMF, (G) radiation flux, (H) non-dimensional vorticity vector, and (I) non-dimensional axial vorticity vector".

It can be seen in "Figure 2(A) that the reduced radial component of fluid velocity, denoted by $\frac{u}{u_1}$, reductions as they move from the shock front to the inner expanding surface when there is a magnetic field present. On the other hand, when there is no magnetic field present, the reduced radial component of fluid velocity decreases near the shock front and rises near the inner expanding surface".

As shown in Figure 2(B)-(I), "as they move from the shock front to the inner expanding surface, there is a decrease in the azimuthal component of fluid velocity $\frac{v}{v_1}$, density $\frac{\rho}{\rho_1}$, pressure $\frac{p}{p_1}$, and radiation flux $\frac{j}{j_1}$, but an increase in the axial component of fluid velocity $\frac{w}{w_1}$, AMF $\frac{h}{h_1}$, and both the azimuthal and axial components of the vorticity vectors l_θ and l_z, respectively".

As a result, "the gas's SW production is less affected by an increase in its adiabatic exponent. Additionally, the fluctuation in the value of the adiabatic index has a greater impact on the flow variables when the initial magnetic field is either weak or absent. The effect of a rise in the value of M_A^{-2} This occurs when the intensity of the magnetic field increases".

"In order to reduce the shock strength, it is necessary to raise the value of β, as shown in Table I.

(i) To lower the value of η_p, which means to measure the distance among the inner expanding surface and the shock front. From a physical point of view, this indicates that the gas behind the shock is fewer compressed, which implies that the shock strength is reduced (see Table I);

(ii) In order to enhance the flow variables $\frac{v}{v_1}$ and $\frac{\rho}{\rho_1}$, while simultaneously decreasing the flow variables $\frac{u}{u_1}, \frac{w}{w_1}, \frac{h}{h_1}, \frac{j}{j_1}$; , as well as l_θ and l_z at any point in the f low-field behind the shock front (refer to Figure 1(A)–(C), (E) (F), and (H)–(I)).

(iii) To raise the decreased pressure $\frac{p}{p_1}$ but the reverse behavior is seen for the reduced radiation flux $\frac{j}{j_1}$ in the flow field behind the shock in general (refer to Figure 2(D) and (G) for more information)".

Therefore, the existence of a magnetic field produces an impact that is similar to a disappearing SW. A comparative impact on the thickness, the azimuthal and pivotal part of liquid speed, and the vorticity vector has likewise been seen when the strength of the encompassing attractive field or the adiabatic type of the gas increments. However, these parameters behave in opposite ways when it comes to the radial component of fluid velocity and radiation flux.

Table I: The density ratio $\beta(= \rho_1/\rho_2)$ varies over the shock front and inner expanding surface at various M_A^{-2} with $\gamma = \frac{5}{3}$.

M_A^{-2}	β	**Inner expanding surface position** $\eta_p \left(= \frac{r_p}{R}\right)$	
		$t/t_0 = 1$	$t/t_0 = 2$
0	0.260000	0.757200	0.831734
0.01	0.261040	0.314055	0.323670
0.02	0.271706	0.213770	0.209205
0.05	0.301950	0.149961	0.135350
0.1	0.348383	0.131740	0.113664

Table II: The density ratio $\beta(= \rho_1/\rho_2)$ varies over the shock front and inner expanding surface at various M_A^{-2} with γ

1	γ	β	**Inner expanding surface position** $\eta_p \left(= \frac{r_p}{R}\right)$		
			$t/t_0 = 1$	$t/t_0 = 1.5$	$t/t_0 = 2$
0	$\frac{4}{3}$	0.142860	0.890958	0.907762	0.918540
	$\frac{5}{3}$	0.260000	0.757200	0.803697	0.831734
0.02	$\frac{4}{3}$	0.185151	0.281775	0.267486	0.258403
	$\frac{5}{3}$	0.271706	0.213770	0.210653	0.209205
0.05	$\frac{4}{3}$	0.232801	0.215954	0.203646	0.195926
	$\frac{5}{3}$	0.301950	0.149961	0.140657	0.135350
0.1	$\frac{4}{3}$	0.296399	0.185652	0.173648	0.166134
	$\frac{5}{3}$	0.348383	0.131740	0.123795	0.113664

5. Conclusion

This inquiry focuses on the self-similar flow that occurs when a strong exponential cylindrical shock wave passes through a rotating axisymmetric ideal gas, as well as the isothermal flow in the presence of an axial magnetic field. The SW is pushed out by a piston that moves in time in accordance with an exponential law. Even though the problem of explosion in rotating conducting media is the primary focus of this investigation, the approach and analysis that have been provided can be applied to a wide range of other physical systems that have non-linear hyperbolic partial differential equations. The study of flare-produced shocks in the solar wind, nucleus explosions, pressurized vessel ruptures, and the central regions of star burst galaxies, as well as the description of shocks in supernova explosions can all benefit from SWs in axisymmetric, perfectly rotating gas with a decreasing initial density and magnetic field. This study may lead to the following conclusions:

- The shock strength increases with time in the absence of a magnetic field; However, when a magnetic field is present in general, the opposite behavior occurs.
- SW dissipation is caused by "magnetic fields the adiabatic exponent and magnetic field have no effect on shock intensity in rotational media". However, as the adiabatic exponent rises, the shock's influence increases in a non-rotating medium with a magnetic field.
- When the "time $(^{t}/_{t_0})$, is less than or equal to 1.5, the density and pressure in both rotating and non-rotating media drop as a function of time $(^{t}/_{t_0})$".
- It is essential to keep in mind that the wave's total energy in the disturbed zone fluctuates over time and is not constant. Additionally, the SW's velocity fluctuates.

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